

## Interstellar molecules

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The large number of molecules, free radicals and ions detected in the interstellar space are given with their column densities. The peculiar conditions to which these molecules are submitted, viz. very low density, relatively large radiation field and very low temperature are described. The suggested processes for the formation of interstellar molecules under these conditions are given with their rate coefficients. The importance of these molecules to space communication is described and absorption of interstellar OH and NH<sub>3</sub> molecules for three resonant frequencies are calculated. Also, the formations of living matters in the interstellar space from constituent atoms in the presence of intense heat, ultra-violet ray, electric discharge or ionising radiations are given.

### 1. INTRODUCTION

The research of interstellar atoms and molecules is at present primarily empirical. Its physics is not known. And, because of the peculiar conditions of interstellar space to which these molecules are submitted—very low density, relatively large radiation field and very low temperature—, it appears that very few molecules can be formed in this region by direct combination of atoms in the gaseous state. Their formation and destruction involve new fields of chemistry. Yet these molecules present exciting problems to astrophysics and cosmology. Again, they are of considerable interest to space communication engineers and of concern to bioscientists.

### 2. DETECTION OF INTERSTELLAR MOLECULES

Earlier in 1940 CN, CH and CH<sup>+</sup> were detected as narrow lines in small clouds having 10-100 atoms per cm<sup>3</sup> (Table 1). Their absorption lines were observed in the spectra of stars. Again, H<sub>2</sub> and CO were detected from satellites by their absorption in the far ultraviolet region. However, most of the interstellar molecules were detected by absorption of rotational lines of these molecules in the millimeter region using radio telescope at National Radio Astronomy Observatory, Kitts Peak, Arizona, U.S.A. These molecules, which are about forty in number, detected upto now by radio telescopes are given in Table 2 with the year of discovery. The tables show that molecules having upto 9 atoms comprising of H, C, N, O, S and Si are present in the interstellar space. It should further be noted that N<sub>2</sub> is not detected but expected to be present, NO is searched for but not detected. Again C<sub>2</sub><sup>+</sup> is not detected.

Table 1. Interstellar molecules detected by means other than radiotelescopes

Discovery	Molecule	Transition	$\lambda(\text{\AA})$
1972	H <sub>2</sub>	$1\Sigma_u^+ \rightarrow 1\Sigma_g^+$	1100
Just before 1940	CH	$2\Sigma^+ \rightarrow 2\pi$	3143
		$2\Sigma^- \rightarrow 2\pi$	3886
		$2\Lambda \rightarrow 2\pi$	4300
..	CH <sup>+</sup>	$1\pi \rightarrow 1\Sigma^+$	3579
..	CN	$2\Sigma^- \rightarrow 2\Sigma^-$	3876

Table 2. Interstellar molecules detected by radio telescopes

Discovery	Molecule	Rational Quantum Numbers	$\nu$ (GHZ)
1963	<sup>16</sup> OH( $2\pi_{3/2}$ )	$J = 3/2$	1.665
1969		$= 5/2$	6.035
1970		$= 7/2$	13.441
1968	<sup>16</sup> OH( $2\pi_1$ )	$J = 1/2$	4.766
1969		$= 5/2$	8.136
1966	<sup>18</sup> OH( $2\pi_{3/2}$ )	$J = 3/2$	1.637
1968	<sup>14</sup> NH <sub>3</sub> (para)	( $J,K$ ) 1,1	23.694
	(para)	$= 2, 2$	23.723
	(ortho)	$= 3, 3$	23.870
	(para)	$= 4, 4$	24.139
	(ortho)	$= 6, 6$	25.056
1969	H <sub>2</sub> <sup>16</sup> O (ortho)	$J_{k-1}K_1 = 5_{23}-6_{16}$	22.235
1969	H <sub>2</sub> <sup>12</sup> C <sup>16</sup> O(ortho)	$J_{k-1}K_1 = 1_{11}-1_{10}$	4.830
		$= 2_{12}-2_{11}$	14.488
		$= 3_{13}-3_{12}$	28.974
1971	H <sub>2</sub> <sup>12</sup> C <sup>16</sup> O(ortho)	$= 1_{11}-2_{12}$	140.839
	H <sub>2</sub> <sup>12</sup> C <sup>16</sup> O (para)	$= 1_{01}-2_{02}$	145.603
	H <sub>2</sub> <sup>12</sup> C <sup>16</sup> O (ortho)	$= 1_{01}-2_{11}$	150.498
1969	H <sub>2</sub> <sup>13</sup> C <sup>16</sup> O(ortho)	$= 1_{11}-1_{10}$	4.593
1970	<sup>12</sup> C <sup>16</sup> O	$J = 0-1$	115.271
1971	<sup>13</sup> C <sup>16</sup> O	$= 0-1$	110.201
1971	<sup>12</sup> C <sup>18</sup> O	$= 0-1$	109.782

Table 2 (Contd.)

Discovery	Molecule	Rational Quantum Numbers	$\nu$ (GHZ)
1970	$\text{H}^{12}\text{C}^{16}\text{O}^{16}\text{OH}$	$J_{k-1}K_1 = 1_{11}-1_{10}$	1.639
1971	$^{12}\text{C}^{32}\text{S}$	$J = 2-3$	146.969
1971	$^{28}\text{Si}^{16}\text{O}$	$J = 2-3$	130.268
1971	$^{12}\text{CH}_3^{12}\text{C}_2\text{H}$ (ortho)	$(JK) = 4.0-5.0$	85.457
1971	$\text{H}^{14}\text{N}^{12}\text{C}^{16}\text{O}$	$J_{k-1}K_1 = 3_{03}-4_{01}$ $= 0_{00}-1_{01}$	87.925 21.982
1971	$^{16}\text{O}^{12}\text{C}^{32}\text{S}$	$J = 8-9$	109.463
1971	$^{14}\text{NH}_2\text{H}^{12}\text{C}^{16}\text{O}$	$J_{k-1}K_1 = 2_{12}-2_{11}$	4.619
1971	$\text{CH}_3\text{HCO}$	$= 1_{10}-1_{11}$	1.065
1917	$^{12}\text{CH}_3^{12}\text{C}^{14}\text{N}$ (ortho)	$(JK) = 5, 0-6, 0$	110.384
	(para)	$= 5, 1-6, 1$	110.381
	(para)	$= 5, 2-6, 2$	110.375
	(ortho)	$= 5, 3-6, 3$	110.364
	(para)	$= 5, 4-6, 4$	110.349
	(para)	$= 5, 5-6, 5$	110.330
		$= 4, 1-4, 2$	24.933
1974	$\text{CH}$	$J_k = 1/2$	3.349
1973	$^{12}\text{C}^{17}\text{O}$	$= 0-1$	112.359
1970	$^{12}\text{C}^{14}\text{N}$	$J = 0-1$	113.492
1970	$\text{H}^{12}\text{C}^{14}\text{N}$	$J = 0-1$	88.632
1970	$\text{H}^{13}\text{C}^{14}\text{N}$	$J = 0-1$	88.339
1970	$\text{H}^{12}\text{C}_3^{14}\text{N}$	$J = 0-1$	9.098
1970	$^{12}\text{CH}_3^{16}\text{OH}$	$J_{k-1}K_1 = 1_{11}-1_{10}$ $(JK) = 4, 1_0-4, 2$ $= 5, 1-5, 2$ $= 6, 1-6, 2$ $= 7, 1-7, 2$ $= 8, 1-8, 2$ $= 4, 1-4, 2$	0.834 24.933 24.959 25.018 25.125 25.294 24.933
1973	$^{32}\text{S}^{16}\text{O}$	$J_1 = 2_1-3_2$ $= 3_2-4_3$	99.3 138.2
1975	$\text{H}_2\text{CS}$	$4_{13}-4_{14}$	10.46
1975	$\text{C}_2\text{H}$		87.317 87.328 87.402
1975	$\text{H}_2\text{C} = \text{CH}-\text{CN}$	$2_{11}-2_{12}$	1.372
1975	Methanol	$3_1-3_1$	5.005
1975	$\text{CH}_3\text{CH}_2\text{OH}$	606-515 414-303 515-404	85.26 90.11 104.8
1976	$\text{HC O O H}$	$2_{11}-2_{12}$	4.9
1975	$\text{HCOO CH}_3$	$1_{10}-1_{11}$	1.61
1975	$\text{CH}_3\text{NH}$	$2_{02}-2_{01}$	88.67

The column densities of interstellar molecules are given in Table 3. This table shows that  $H_2$  is most abundant having a column density of  $10^{23} \text{cm}^{-2}$  (compare for  $H$ , it is  $10^{21} \text{cm}^{-2}$ ). Next most abundant molecule in the interstellar space is  $CO$  with a column density of  $10^{19} \text{cm}^{-2}$ .

Table 3. Column densities in the direction of Sagittarius  $B_2$  and the Orion Nebula

Molecule	Column density in SgrB <sub>2</sub> ( $\text{cm}^{-2}$ )	Column density in Orion ( $\text{cm}^{-2}$ )
$H_2$	$> 10^{23}$	$2 \times 10^{25}$
$OH$	$> 5 \times 10^{16}$	—
$CO$	$\sim 10^{19}$	$\sim 10^{18}$
$CN$	Not detected	$\sim 10^{15}$
$CS$	$\sim 10^{14}$	$2 \times 10^{13} - 5 \times 10^{14}$
$SiO$	$\sim 4 \times 10^{13}$	Not detected.
$H_2O$	—	—
$HCN$	Not determined	$\sim 10^{15}$
$OCS$	$\geq 3 \times 10^{15}$	Not detected
$NH_3$	$\geq 10^{17}$	Not detected
$H_2CO$	$\sim 2 \times 10^{15}$	$\sim 3 \times 10^{14}$
$HNCO$	Not determined	Not detected
$HC_3N$	$\sim 2 \times 10^{16}$	Not detected
$HCOOH$	$10^{13} - 3 \times 10^{15}$	Not detected.
$CH_3OH$	—	$\sim 5 \times 10^{16}$
$CH_3CN$	$\sim 2 \times 10^{14}$	Not detected
$CH_3C_2H$	Not determined	Not detected
$NH_2HCO$	Not determined	Not detected.
$CH^+$	10 cm in the direction of $\zeta \text{oph}$	
$H$	$10^{21}$ same as for $CH^+$	

$H^+ \sim 10^{15} \text{cm}^{-2}$  is a source of protons in H I region

### 3. UNFAVOURABLE PHYSICAL CONDITIONS OF INTERSTELLAR SPACE

The difficulties of understanding the presence of molecules in the interstellar space are many. The molecules in this region are immersed in a field with a very low blackbody temperature (Table 4). This became apparent after the discovery of the universal microwave 2.7°K background radiation whose molecular lines are seen in emission in the mm wavelength range. Because of the relative low kinetic temperature, interstellar molecules are formed in the low electronic and vibrational levels. They occupy only the first few levels in the rotational ladder. H<sub>2</sub>O and possibly methyl alcohol show effects corresponding presumably to collisional pumping.

Table 4. Interstellar and intergalactic space

Region of Space	Density (atom cm <sup>-3</sup> )	Magnetic field (Gauss)	Blackbody Temp (°K)
Interstellar	1	10 <sup>-5</sup> - 10 <sup>-6</sup>	2.7
Intergalactic	10 <sup>-5</sup>	10 <sup>-7</sup> - 10 <sup>-8</sup>	2.7

The interstellar space has low density as shown in Table 3. In the tenuous atmosphere the formation of polyatomic molecules cannot occur in the gas phase. It is suggested that, they are formed in or on the surface of interstellar grains by reactions between atoms and free radicals or may even be catalysed by ions. However, interstellar grains have so low temperature that the molecules have no vapour pressure and cannot normally evaporate from the grain surface. A number of possible mechanisms is advocated to account for the release of molecules from the grain into the gas phase. For example, ions and electrons may recombine on small grains releasing energy whereby micrograins may be locally heated so that molecules may be evaporated.

The interstellar space is crossed by relatively large radiation field—cosmic rays, stellar radiations, and of electrons, ions and neutral particles. Ionization and decomposition of interstellar molecules may occur abundantly by cosmic radiations. However, thick interstellar clouds are detected. They shield molecules against photodissociation by stellar ultraviolet radiations. As a result, molecules will be much slower decomposed behind these clouds as compared with other regions in the interstellar space and the rate of formation of molecules is sub-

tentially enhanced. Fig 1 shows clouds of CH in the interstellar space. Again, some of the important collisional processes, such as charge exchange involving ions and neutral particles cannot occur at the low temperature of interstellar space.

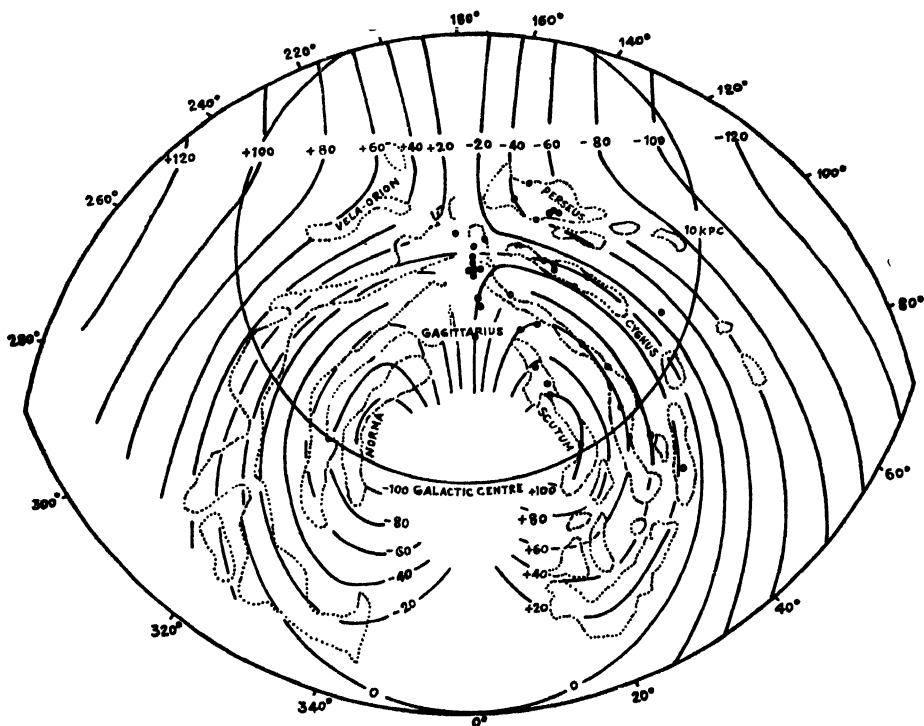


Fig. 1. Location of CH clouds in the interstellar space.

#### 4. FORMATION OF INTERSTELLAR MOLECULES

A few suggestions for the formation of interstellar molecules is made. They are given in Table 5 with the rate coefficients. However, in order to know precisely their formation and destruction processes, laboratory data and theoretical calculations of collisional cross section are needed.

#### 5. IMPORTANCE OF INTERSTELLAR MOLECULES TO SPACE COMMUNICATION

For space communication radiations in the microwave region (1-10GHz) is used. For such communications absorption of this radiation in the planetary atmospheres should be known. The method of calculating absorption in the earth's atmosphere was developed by Ghosh and Malaviya (1961). It can be extended for obtaining absorption of this radiation in the other planetary atmospheres. The absorption in the intervening interplanetary space can not be obtained before as the presence of interstellar molecules and their distributions are not known.

Table 5. Certain processes suggested for the formation of interstellar molecules

Reaction	Rate coefficient (cm <sup>3</sup> sec) <sup>-1</sup>	Remark	Investigator
CH <sup>+</sup> + e → CH + hν		Adequate quantity of CH cannot be produced by this reaction (Smith <i>et al</i> 1973)	Krauss and Julianne
$\left\{ \begin{array}{l} \text{CH}^+ + \text{H}_2 \rightarrow \text{CH}_3^+ + \text{H} \dots (1) \\ \text{CH}_3^+ + e \rightarrow \text{C} + \text{H}_2 \quad \left[ \rightarrow (2) \right] \\ \quad \quad \quad \rightarrow \text{CH} + \text{H} \\ \text{CH}_3^+ + e \rightarrow \text{CH} + \text{H}_2 \quad \left[ \rightarrow (3) \right] \\ \quad \quad \quad \rightarrow \text{CH}_2 + \text{H} \end{array} \right.$	$\begin{array}{l} 10^{-9} \\ 3 \times 10^{-7} \\ 3 \times 10^{-7} \end{array}$	<p>If the product of the reaction (3) is mainly CH<sub>2</sub>, it is likely that photodissociation produces CH in an appreciable fraction of the dissociation (CH<sub>2</sub> + hν → CH + H). For a typical cross section for CH<sub>2</sub>, photodissociation is at least comparable with other destruction mechanisms (CH<sub>2</sub> + C<sup>+</sup> → C<sub>2</sub>H<sup>+</sup> + H) in the interstellar clouds of low to moderate optical shielding (E<sub>2</sub>, ν ≲ 0.5 mag); which of the possible products dominates in the reaction (3) will then be relatively unimportant. In summary, we provisionally assume that every reaction (1) ultimately leads to CH. (Watson 1974)</p>	<p>Franklin <i>et al</i> (1973)</p> <p>Leu <i>et al</i> (1973a, b, c)</p> <p>"</p>
C <sup>+</sup> + H <sub>2</sub> → CH + H <sup>+</sup> - 3.35ev	10 <sup>-11</sup>	H <sub>2</sub> is vibrationally excited with ν' > 9	Howe and Herzberg (1959)
C <sup>+</sup> + H <sub>3</sub> <sup>+</sup> → CH + H <sup>+</sup>	10 <sup>-9</sup>	This reaction is endothermic for ν' > 9	Stecher and William (1972)
C <sup>+</sup> + H → CH <sup>+</sup> + hν	10 <sup>-17</sup>		Smith <i>et al</i> (1973)
CH + hν → CH <sup>+</sup> + e	Rate = 4 × 10 <sup>-11</sup> sec <sup>-1</sup>	This process forming CH <sup>+</sup> by photo-ionisation of CH produces inadequate amount of CH in typical situation because of the fast loss of CH <sup>+</sup> . Only radiative association C <sup>+</sup> , H and H <sub>2</sub> <sup>+</sup> need be considered.	Smith <i>et al</i> (1973)

Table 5 (contd.)

This reaction is endothermic for $v' \geq 1$			
$C^+ + H_2 \rightarrow CH^+ + H - 0.4\text{eV}$	$10^{-11}$	Herzberg and Howe (1959)	
$C^+ + H_3^+ \rightarrow CH^+ + H$	$10^{-9}$	Stecher and Williams (1972)	
$CH^+ + H_3 \rightarrow CH_3^+ + H$	$10^{-9}$	Franklin <i>et al</i> (1963)	
$C^+ + H_2 \rightarrow CH_2^+ + h\nu$	$4 \times 10^{-17}$	Fehsenfeld <i>et al</i> (1974)	
$CH_2^+ + H_3 \rightarrow CH_3^+ + H$	$10^{-9}$	Franklin <i>et al</i> (1963)	
$CH^+ + O \rightarrow CO + H^+$	$10^{-7}$	Solomon <i>et al</i> (1972)	
$CH^+ + e \rightarrow C + H$	$10^{-7}$	Krauss and Julienne (1973)	
$CH + h\nu \rightarrow C + H$	Rate = $11 \times 10^{-11}\text{sec}^{-1}$	Smith <i>et al</i> (1973)	
$CH_2^+ + e \rightarrow C + H_2$	$3 \times 10^{-7}$	Watson (1974)	
$C^+ + H_2 \rightarrow C + H_2^+ - 4.17\text{eV}$	$10^{-11}$	Herzberg and Howe (1959)	
$CH^+ + h\nu \rightarrow C^+ + H$	Rate = $10^{-12}\text{sec}^{-1}$	Smith <i>et al</i> (1973)	
$CH_3^+ + e \rightarrow CH_3 + H$	$3 \times 10^{-7}$	Watson (1974)	
$CH + C^+ \rightarrow C_2^+ + H$	$10^{-9}$	Solomon & Klemperor (1972)	
$H_3^+ + He \rightarrow HeH^+ + H$		Von Koch & Friedman (1963)	
$H + H^- \rightarrow H_2 + e$	$1.3 \times 10^{-9}$ (at 300°K)	Ferguson <i>et al</i> (1969)	
$N_2^+ + Na \rightarrow Na^+ + N_2$	$5.8 \times 10^{-10}$ (at 300°K)	"	
$O^+ + N_2 \rightarrow NO^+ + N$	$1.2 \times 10^{-12}$ for $v' = 0$	"	
	$1.2 \times 10^{-12}$ for $= 1$	"	
	$50 \times 10^{-12}$ for $= 2$	"	
	$120 \times 10^{-12}$ for $= 3$	"	

H<sub>2</sub> is vibrationally excited with  $v'' \geq 12$ 

Endothermic by 1.1. e.v.



From the information of interstellar molecules and their column densities given in Tables 2 and 3, it is now possible to obtain the absorption for microwaves used for space communication.

The microwave absorption  $\gamma$  in  $dB/km$  is calculated from the formula deduced by the above investigators,

$$\gamma = \gamma_0 \frac{\Delta\nu^2}{(\nu_0 - \nu)^2 + \Delta\nu^2} \frac{\nu^2}{\nu_0^2}$$

where  $\gamma_0 = 10^6 \alpha_0 \log_{10} e$  is the value of  $\gamma$  at the peak resonance frequency  $\nu_0$ .

$\Delta\nu$ —half width of the absorption line. The absorption coefficient at the peak resonance frequency is given by

$$\alpha_0 = \frac{8\pi^2 \nu_0^2 N f}{3ckT} (\mu_{ij})^2 \frac{1}{\Delta\nu}$$

$$\alpha_0 \propto \frac{Nf}{\Delta\nu T} \text{ since } \frac{8\pi^2 \nu_0^2}{3ck} (\mu_{ij})^2 \text{ is a constant factor,}$$

where

- $N$ — total no. of molecules per ml.
- $f$ — fraction of total number of molecules in the lower transition state
- $T$ — the absolute temperature in  $^{\circ}K$ ,
- $\mu_{ij}$ — matrix element of the component of the dipole moment for the transition  $i \rightarrow j$ .

The absorptions for interstellar OH radical at 7.8 GHz peak frequency and  $NH_3$  for peak frequencies 23.87 and 24.14GHz have been calculated using the above formula and with the following data. The calculated absorptions are plotted against frequencies in Figs 2, 3 and 4. For comparison the absorption curve for terrestrial atmosphere as obtained by Ghosh and Malaviya is given in Fig. 5.

OH

$$\nu_0 = 7.8 \text{ GHz}$$

$$N = 10^{-8} \text{ cm}^{-3}$$

$$f = 1$$

$$\mu_{ij} = \mu = 1.54 \times 10^{18} \text{ e.s.u./cm}$$

$$\Delta\nu = 25 \text{ MHz}$$

$$k = 1.37 \times 10^{-16}$$

$$T = 300^{\circ}K$$

$$C = 3 \times 10^{10} \text{ cm/s}$$

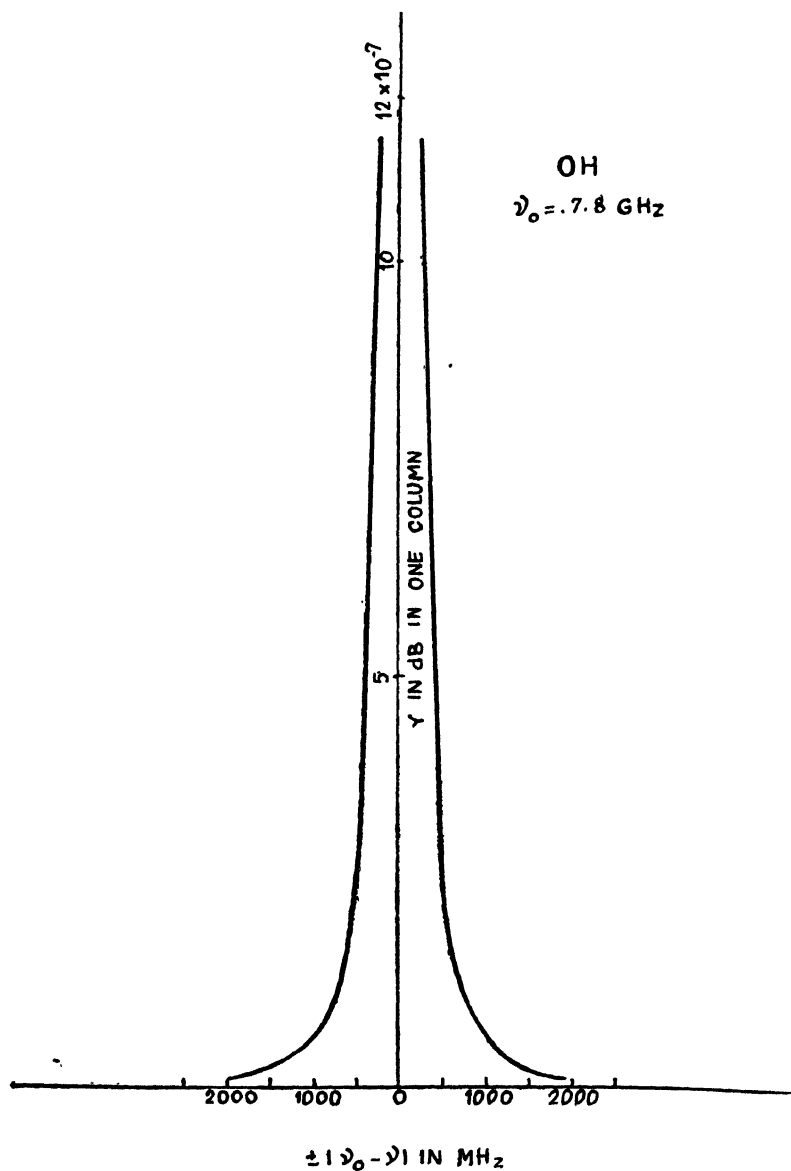


Fig. 2. Absorption for interstellar OH radical at 7.8 GHz peak frequency.

$NH_3$

$$\nu_0 = 23.87, 24.14 \text{ GHz}$$

$$N = 2 \times 10^{-4} \text{ cm}^{-3}$$

$$f = 1$$

$$\mu_f = \mu = 1.47 \times 10^{-18} \text{ e.s.u./cm.}$$

$$k = 1.37 \times 10^{-18}$$

$$T = 300^\circ \text{K}$$

$$\Delta\nu = 25 \text{ MHz.}$$

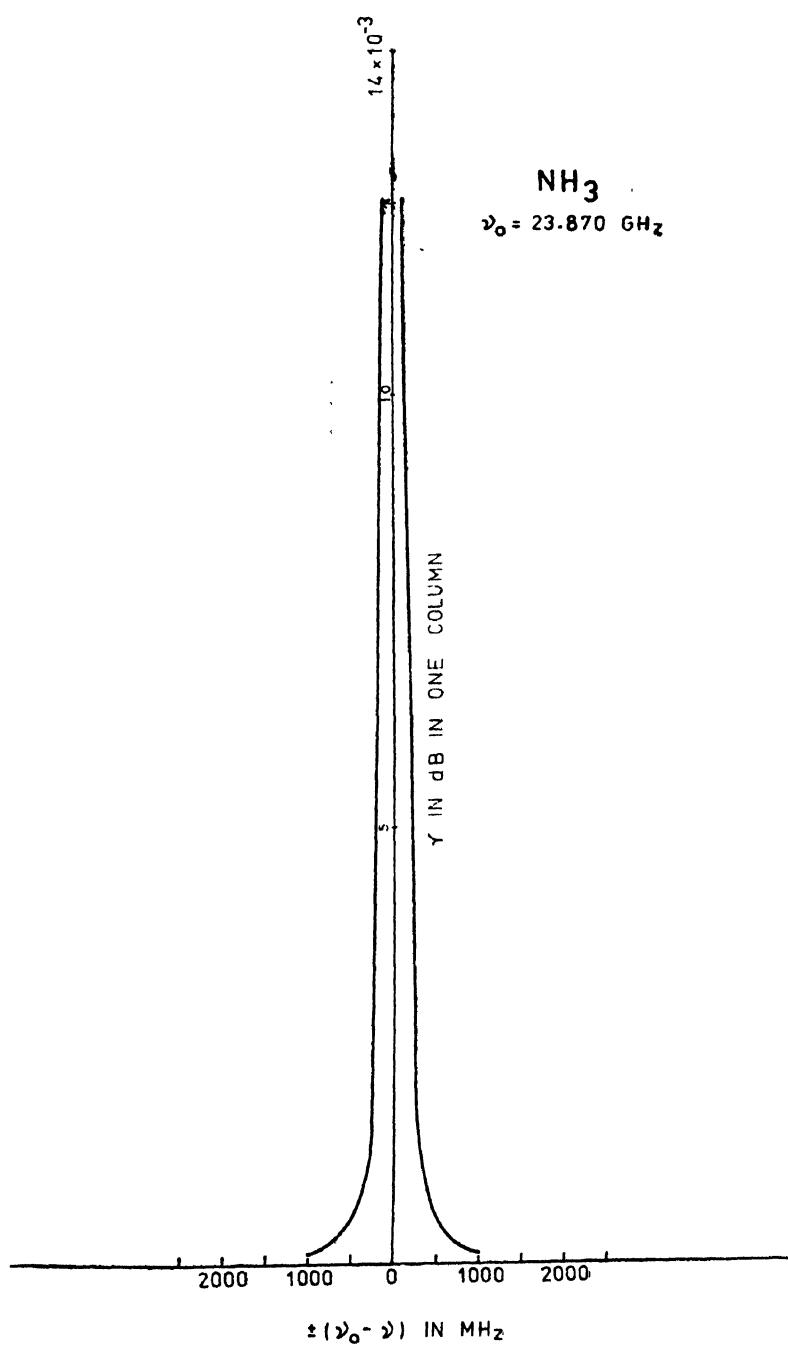


Fig. 3. Absorption for interstellar  $\text{NH}_3$  molecules at peak frequency 23.87 GHz.

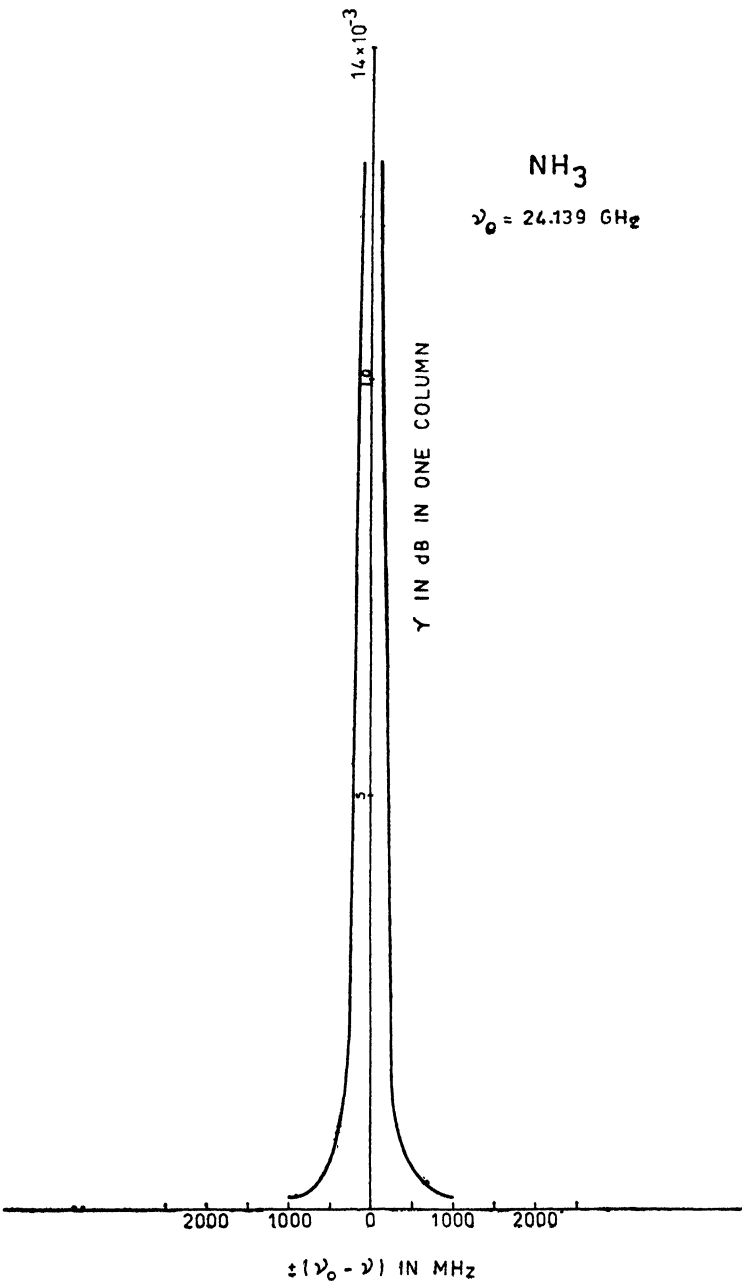


Fig. 4. Absorption for interstellar  $\text{NH}_3$  molecules at peak frequency 24.14 GHz.

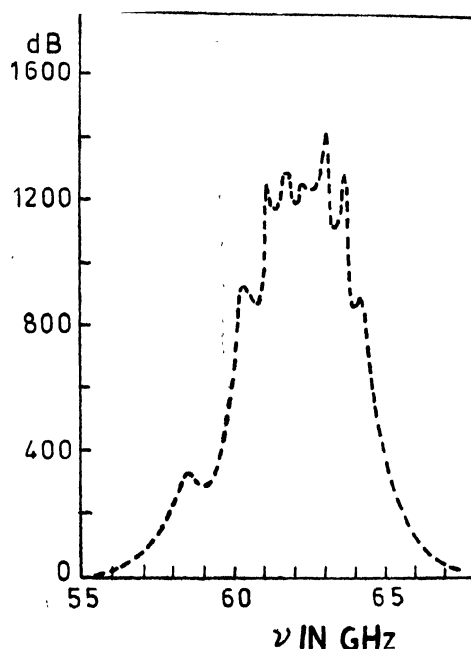


Fig. 5. The microwave absorption for terrestrial atmosphere as obtained by Ghosh & Malaviya.

#### 6. FORMATION OF LIVING MATTERS IN INTERSTELLAR SPACE

As already mentioned, molecules consisting of H, C, N, O, S and Si atoms occur in the interstellar space. From these atoms the basic elements out of which living matters are formed may occur (Table 6).

Table 6. Formation of living matters in the interstellar space

$H \rightarrow H_2$	$\left\{ \begin{array}{l} \text{Ultraviolet light} \\ \text{Electric discharge} \\ \text{Ionizing radiation} \\ \text{Heat} \end{array} \right\}$	Aminoacid $\rightarrow$ Protein
$C \rightarrow CH_4$		Purines
$N \rightarrow NH_3$		Pyrimidines $\rightarrow$ Nucleic acid
$O \rightarrow O_2$		Carbohydrates $\rightarrow$ Nucleic acid

Out of three complex organic molecules involving amino-acids, carbohydrate etc are expected to be formed. However, it is difficult to check this conclusion because the r.f. spectrum of these molecules are generally unknown.

There are many uncertainties and conjectures in the formation and destruction of interstellar molecules and a lot more molecular physics and chemistry is required before we can obtain a fairly accurate information of the interstellar molecules.

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